Module 2 Summary

# Response to feedback

**If you are resubmitting**, include a statement outlining the changes you have made to your submission. This section can be short but should be precise. It is a good idea to quote the feedback you are responding to.

**If this is your first submission**, include a statement about what part of the lesson review you would most like to receive feedback (and why). Your tutor will take this into consideration when reviewing your work, although they may choose to give you feedback on a different thing if they think it’s more appropriate.

# Module Learning Objectives

I certify that I achieved the following learning objectives for the module (these objectives can be found in the introduction of the module):

* …
* …

# Key Definitions and Theorems

Matrix addition, scalar multiplication and transpose

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| Textbook Ref | Theorem/Definition statement | Learning Notes |
| Definition 2.1  Matrix Addition | If A and B are matrices of the same size, their sum A+B is the matrix formed by adding corresponding entries, i.e. for the entries *aij*and *bij*in A and B, the entry in the matrix A+B will be *aij*+ *bij* |  |
| Definition 2.2 Matrix Scalar Multiplication | If A is any matrix and *k* is any number, the scalar multiple *k*A is the matrix obtained from A by multiplying each entry of A by *k, i.e. kaij* for all entries. |  |
| Theorem 2.1.1 Properties of matrix addition and multiplication by a scalar | Let A, B, and C denote arbitrary *m*×*n* matrices where m and n are fixed. Let *k* and *p* denote arbitrary real numbers. Then  1. A+B = B+A  2. A+(B+C) = (A+B)+C  3. There is an *m*×*n* matrix 0, such that 0+A = A for each A  4. For each A there is an m×n matrix, −A, such that A+(−A) = 0  5. k(A+B) = kA+kB  6. (k+ p)A = kA+ pA  7. (kp)A = k(pA)  8. 1A = A | 1. commutativity  2. associativity  3. additive identity element  4. additive inverse  5. distributivity for scalar times sum of two matrices  6. distributivity for sum of two scalars times a matrix  7. associativity with scalars  8. multiplicative scalar identity element |
| Definition 2.3 Transpose of a matrix | If A is an *m*×*n* matrix, the transpose of A, written AT, is the *n*×*m* matrix whose rows are just the columns of A in the same order. |  |
| Theorem 2.1.2 Properties of transpose | Let A and B denote matrices of the same size, and let *k* denote a scalar.  1. If A is an *m×n* matrix, then AT is an *n×m* matrix.  2. (AT )T = A.  3. (*k*A)T = *k*AT.  4. (A+B)T = AT +BT. |  |

Matrix-vector multiplication

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| Textbook Ref | Theorem/Definition statement | Notes |
| Definition 2.4 The set ℝn | Let **ℝ** denote the set of all real numbers. The set of all ordered n-tuples from **ℝ** is denoted by **ℝ**n |  |
| Definition 2.5  Matrix-vector multiplication | Let be an *m×n* matrix, written in terms of its columns . If is any n-vector (i.e., **x** ∈ **ℝ**n), the product A**x** is defined to be the m-vector in **ℝ**m given by: |  |
| Theorem 2.2.2 Properties of matrix-vector multiplication | Let A and B be *m×n* matrices, and let **x** and **y** be n-vectors in **ℝ**n. Then:  1. A(**x**+**y**) = A**x**+A**y**.  2. A(*a***x**) = *a*(A**x**) = (*a*A)**x** for all scalars *a*.  3. (A+B)**x** = A**x**+B**x**. |  |
| Definition 2.6 Dot product in ℝn | If (*a*1,*a*2,…,*an*) and (*b*1,*b*2,…,*bn*) are two ordered n-tuples in **ℝ**n, their dot product is defined to be the number  *a*1 *b*1 + *a*2 *b*2 +… + *an bn*  Obtained by multiplying corresponding entries and adding the results. |  |
| Theorem 2.2.3 General solution for non-homogenous systems | Suppose **x**1 is any particular solution to the system A**x** = **b** of linear equations. Then every solution **x**2 to A**x** = **b** has the form  **x**2 = **x**0 + **x**1  for some solution **x**0 of the associated homogeneous system A**x** =**0**. |  |
| Theorem 2.2.4 Relationship between rank of augmented matrix and consistency | Let A**x** = **b** be a system of equations with augmented matrix [A | **b** ]. Let rank A = *r*.  1. rank [A | **b** ] is either *r* or *r* + 1  2. The system is consistent if and only if  rank [A | **b** ] = *r*  3. The system is inconsistent if and only if rank  rank [A | **b** ] = *r* + 1 |  |
| Theorem 2.2.5 Dot product rule | Let A be an *m×n* matrix and let **x** be an n-vector in **ℝ**n. Then each entry of the vector A**x** is the dot product of the corresponding row of A with **x**. |  |
| Theorem 2.2.6  Uniqueness of matrix-vector multiple | Let A and B be *m×n* matrices. If A**x** = B**x** for all **x** in **ℝ**n, then A = B. |  |
| Definition 2.8 Matrix Transformation *T*A | *T*A is called the matrix transformation induced by A, with *T*A : **ℝ**n → **ℝ**m defined by *T*A (**x**) = A**x** for every **x** in **ℝ**n |  |

Matrix multiplication

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| Textbook Ref | Theorem/Definition statement | Notes |
| Definition 2.9 Matrix Multiplication | Let A be an *m×n* matrix, let be an *n×k* matrix, where is column *j* of B for each *j*. The product matrix AB is the *m×k* matrix defined as follows: |  |
| Theorem 2.3.1  Associativity with matrix-matrix-vector multiplication | Let A be an *m×n* matrix and let B be an *n×k* matrix. Then the product matrix AB is *m×k* and satisfies  A(B**x**) = (AB)**x** for all **x** in **ℝ**k |  |
| Theorem 2.3.2 Dot Product Rule for matrix-matrix multiplication | Let A and B be matrices of sizes *m×n* and *n×k*, respectively. Then the (i, j)-entry of AB is the dot product of row *i* of A with column *j* of B. |  |
| Theorem 2.3.3 Properties of matrix-matrix multiplication | Assume that *a* is any scalar, and that A, B, and C are matrices of sizes such that the indicated matrix products are defined and I is an identity matrix.  Then:  1. IA = A and AI = A  2. A(BC) = (AB)C.  3. A(B+C) = AB+AC.  4. (B+C)A = BA+CA.  5. *a*(AB) = (*a*A)B = A(*a*B).  6. (AB)T = BTAT |  |

# Summarising your understanding:

* Add notes to the definitions and theorems tables that reflect your understanding of the definition/theorem, how it relates to the learning objectives and things to keep in mind when applying – these notes can be somewhat informal but should be understandable to your OnTrack tutor.

Write a mathematical summary for the module that includes the questions/tasks in the dot points below. Note that the summary should include more than just the questions - we’d recommend having a subheading for each sub-module, and then include the guiding questions where appropriate. You should provide reference to the relevant definitions, theorems and terms in the tables above to help demonstrate your understanding of how the concepts are linked and applied (as you did for Module 1).

* Summarise the different ways of approaching matrix-matrix multiplication (see the video on the unit site) making reference to the relevant definitions above.
* Provide an example that demonstrates item 6 of Theorem 2.3.3 (and in particular that (AB)T does not equal ATBT ).
* Demonstrate Theorem 2.2.3 with an example in matrix form.

# Reflecting on the content:

* Reflect on what you found interesting/difficult in this module, how it relates to previous content covered in this and other units, and which parts helped you to meet each of the learning objectives. This should be written from your personal perspective and not just read as a general summary of the topic.